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Numerical Investigation for Cell Modeling Of Hydro Magnetic Axial Flow over a Cylinder

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Abstract

In this paper we consider the filtration problem across a membrane composed of an aggregate of parallel circular cylinders wherein each cylinder is covered by a concentric porous shell and subject to a radial magnetic field. The cell model is applied so that system is taken as equivalent to a single cylinder encased in the porous shell and enclosed by a concentric cylindrical enveloping surface under axial flow of a conducting fluid subject to a radial magnetic field. Our aim is to evaluate the effect of magnetic field on permeability parameter. Bvp4c tool is used to solve the system of equations. The results are then graphically presented and discussed. The analysis reveals that permeability decreases with Hartmann number M for all values of gap and that increasing the gap increases the permeability for all values of M.

Keyword: Hydro Magnetic.

Introductions

Cell model was advanced by Happel [1] and Kuwabara [2] to obtain in a simple manner the analytical results for the complex problem of flow past a concentrated assemblage of particles. The method is dealt with in the above papers and also in the book Low Reynolds Number Hydrodynamics by Happel and Brenner [3]. The cell method is concerned with the study of slow flow past a swarm of concentrated particles. The essence of the method consists of replacing the swarm by a single particle enclosed in an envelope and the interaction effect of the multitude of particles being accounted by suitable boundary condition at the enveloping surface. The use of this method in our context follows from observing that filtration membrane is composed of an aggregate of tiny particles through a fluid percolates slowly.

Vasin and Flippov [4] used cell model to evaluate the hydrodynamic permeability for a system of sold spherical particles covered with a porous shell. In another paper [5] they investigated the flows in a concentrated media composed of rigid impermeable cylinders covered with a porous layer; both transverse and longitudinal flows were taken up. Amongst similar works, we may mention the contributions of Kirsh [6], [7].

Here we aim to study the controlling factor of electromagnetic Lorentz force when the flid is electrically conducting and flows through a membrane composed of array of parallel circular non conducting cylinders with each cylinder covered by a porous cylindrical layer and subject to radial magnetic field. The flow in the clear fluid is governed by Stokes equation and in the porous medium by Brinkman equation [8] the flow is taken along the axes of the cylinders; the cross flow problems have also been investigated and will be presented in a separate paper.

In a companion paper, Part I of this study of cell model for hydromagnetic axial flow over a cylinder, we have studied the case of uniform transverse magnetic field. But here we introduce an additional parameter in the form of porosity of the porous shell covering each of the constituent impermeable cylinder.

Basic equations

The basic hydromagnetic field quantites are velocity **u´**, pressure p´, magnetic field **B´**, electric field **E´** and current density vector **J´.** Let us consider uniform steady axial flow U_0 along a cylindrical body (composed of a solid core covered by a concentric porous shell) of radius a and subject to radial magnetic field of strength $B_0/r´$. It will be convenient to non-dimensionalize the quantities as follows

$$r' = ar, z' = az, u' = Uu, p' = \frac{\mu U_0}{a}p, \tag{1}$$
$$\boldsymbol{B'} = B_0\boldsymbol{B}, \boldsymbol{E'} = U_0 B_0 \boldsymbol{E}, \boldsymbol{J'} = \sigma\mu_m$$

It will be convenient to non – dimensional form as follows

Stokes Equation

$$0 = -\nabla p + \nabla^2 \mathbf{u} + M^2 \mathbf{J} \text{ x } \mathbf{B} \tag{2}$$
$$\nabla . \boldsymbol{u} = 0 \tag{3}$$

Here, $M^2 = \sigma a^2 B_0^2 / \mu$ is the square of Hartmann number

Brinkman Equation

$$0 = -\nabla p + m\nabla^2\mathbf{u} - s^2\mathbf{u} + M^2\mathbf{J} \text{ x } \mathbf{B} \tag{4}$$
$$\nabla . \boldsymbol{u} = 0 \tag{5}$$

Here m $= \mu_e/\mu$ is the ratio of effective viscosity to viscosity and $s^2 = \frac{a^2 k}{\mu}$ is porosity parameter, k being the porosity coefficient in dimensional Brinkman equation:

Maxwell equations:

$$\nabla\text{x}\mathbf{E} = 0, \ \nabla . \mathbf{E} = \rho_e \tag{6}$$

ρ_e represents the total charge density.

$$\nabla\text{x}\mathbf{B} = R_m\mathbf{J}, \nabla . \boldsymbol{B} = \boldsymbol{0} \tag{7}$$

Here $R_m = \mu_m\sigma Ua$ is magnetic Reynolds. Ohm's law

$$\mathbf{J} = \mathbf{E} + \mathbf{u} \text{ x } \mathbf{B} \tag{8}$$

Continuity equation for the current density vector is

$$\nabla . \mathbf{J} = \boldsymbol{0} \tag{9}$$

Reseler and Sears [9] pointed out that the term **u x B** in Ohm's can be taken to represent a tiny generator or source of e.m.f at any point in the moving fluid. The vector **E** represents the total electric field arising out of internal causes such as separation of charges or polarization and external causes such as charged boundaries of the flow. Thus the electric field cannot be dissociated from the fluid motion; and its value within the fluid element is directly affected by the motion of the element and is taken to be of the order **u x B.** Hence, we conclude that for E to vanish, we must have

$$\nabla . (\mathbf{u} \text{ x } \mathbf{B}) = 0 \tag{10}$$

Framing of the problem

The cell method simplifies the problem to consideration of uniform steady flow in the axial direction, taken along the z- axis enveloped in a cell and subject to a radial magnetic field
[See Figs. (1),(2)]. Thus in the non-dimensional variables the inner cylinder is of unit radius, the enveloping cylinder of radius c = 1/d, the external flow e_z and applied magnetic field e_r/r. It may be seen that except of pressure all quantities are independent of z and functions of r alone. Further, we assume that magnetic Reynolds number R_m is small and find that only z- component of the magnetic field is induced. Also, we take m = 1 but the analysis may be extended to other positive values of m. Thus, the problem reduces to the determination of the differential equations and boundary conditions for the velocity u(r)e_z and induced magnetic field b(r)e_z.
Thus, we take

$$\mathbf{u} = u(r)e_z \tag{11}$$

$$B = \frac{e_r}{r} + b(r)e_z \tag{12}$$

Fig. 1

Fig.2
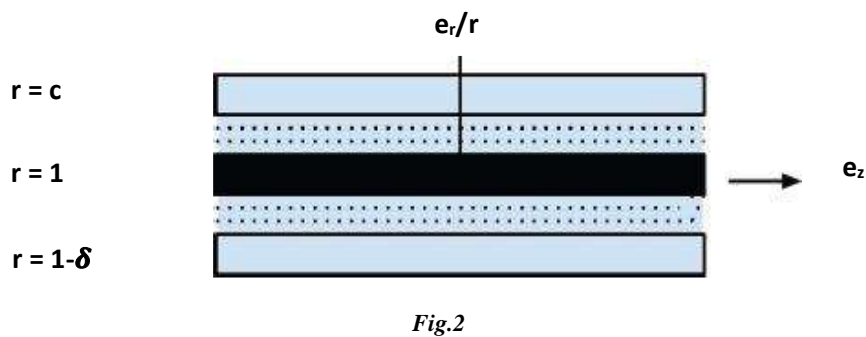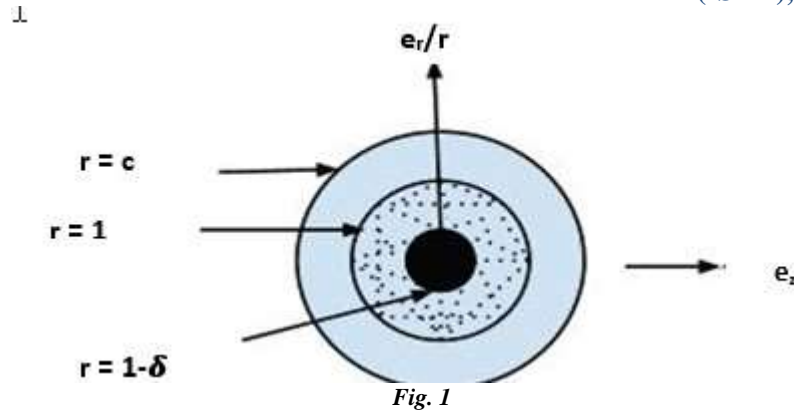
We find that
$$\mathbf{u}x\mathbf{B} = \frac{ue_\theta}{r} \qquad (13)$$
So that
$$\Delta.\mathbf{u}x\mathbf{B} = 0 \qquad (14)$$
Hence, we can take E = 0. We may check that all the hydro magnetic equations are satisfied with u and b given by (11) and (12). Now onwards, we shall designate velocity in the Stokes region II (1 < r < c) by U and in the Brinkman region I(1 - ö < r < 1) by V.

MHD equations for the problem
For the problem in hand we find that the component Stokes and Brinkman equations are expressible as below.
Stokes equation in region II
$$0 = -\frac{\partial p}{\partial r} + \frac{M^2\,Ub}{r} \qquad (15)$$
$$0 = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) - \frac{M^2 U}{r^2} \qquad (16)$$
$$\frac{\partial b}{\partial r} = -\frac{R_m U}{r} \qquad (17)$$
The third of the above equations shows that $\frac{\partial p}{\partial z}$ is constant and this leads to the determination of velocity U. Equation (17) provides the induced magnetic and then the one the pressure as a function of r and z.
Brinkman Equations in region I
$$0 = -\frac{\partial p}{\partial r} + \frac{M^2\,Vb}{r} \qquad (18)$$
$$0 = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) - s^2 V - \frac{M^2 V}{r^2} \qquad (19)$$
$$\frac{\partial b}{\partial r} = -\frac{R_m V}{r} \qquad (20)$$
 It may be noted that in the region I, we continue to write the same symbols for pressure p and induced magnetic field b; this is because pressure gradient $\frac{\partial p}{\partial z}$= P is constant and we are not here evaluating p as we are interested in finding the velocity field and for that equations (16) and (19) together with the boundary conditions are sufficient.

Boundary conditions

At the solid surface r = 1 - δ

No Slip :

$$V = 0 \tag{21}$$

At the interface r = 1:

Continuity of the velocity

$$U(1) = V(1) \tag{22}$$

Continuity of the stress

$$\frac{dV}{dr} = \frac{dU}{dr} \tag{23}$$

Happel, Kuwabara, Kvashnin, Morse-Mehta/Cunningham conditions, all reduce to

$$\frac{du}{dr} = 0 \tag{24}$$

Plots and discussion

The solutions of the two differential equations (16) and (19) of the second order are solved using bvp4c tool in Matlab. In this method equation (16) and equation (19) are converted into first order differential equations and multiboundary condition method has been used to solve the system of equations. The suitable initial guess has been used to solve the set of equations.

The graphs have been plotted for variation of permeability with shell thickness, with Hartmann number and with d. The effect of magnetic field on velocity profile have been plotted in the graph.

The variation of permeability L with porous shell thickness has been shown graphically in Fig.3. The effect has been studied in the absence of magnetic filed and in the presence of magnetic field. The permeability increases in the absence of magnetic field with the increasing porous shell thickness smoothly. But with M = 1.0 the behavior changes, the permeability increases with porous shell thickness but very slowly.

Fig.3: Variation of permeability L with the Porous shell thickness
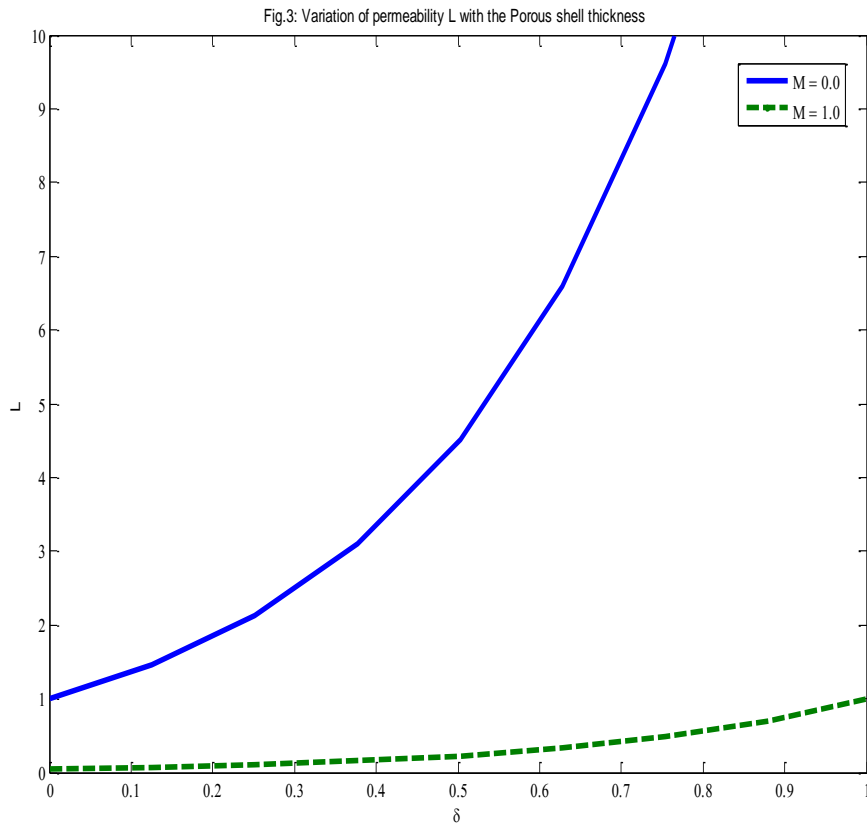
The variation of permeability L with the Hartmann number M has been plotted in Fig. 4 with different outer boundary. The permeability increases smoothly with Hartmann number as well as with the increase of outer boundary. But as the outer boundary increases the effect of magnetic field becomes less significant. The graph has been plotted for different values of c. for c = 2.0, the variation of permeability is very small and does not change significantly with the variation of magnetic parameter M.

For c = 3.0 the values of permeability parameter increases sharply and changes significantly with the variation of Hartmann number M. The same behavior is observable for c = 4.0. The effect of variation in the value of c shows that the importance of cell boundary of the considered model.

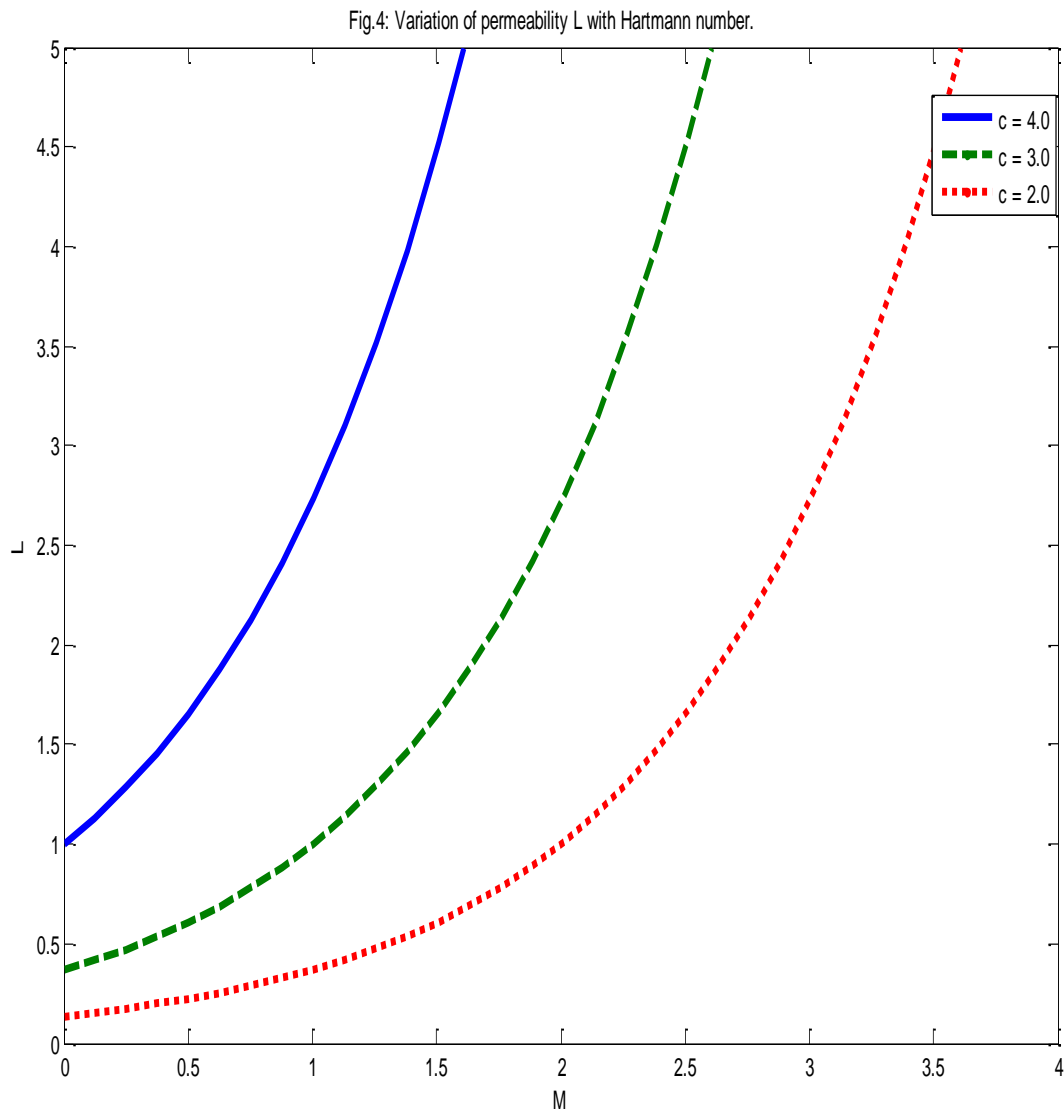Fig.4: Variation of permeability L with Hartmann number.

Fig.5 shows variation of Permeability L with the gap parameter d for different values of Hartmann number. With the increase of gap parameter permeability increases sharply in the absence of magnetic field. But with M = 1.0 permeability increases very slowly with gap parameter. As M increases to 2.0 the effect of gap parameter on permeability is almost negligible.

The effect of Magnetic parameter has been viewed simultaneously with the variation of permeability.

Fig.5: Variation of Permeability L with d
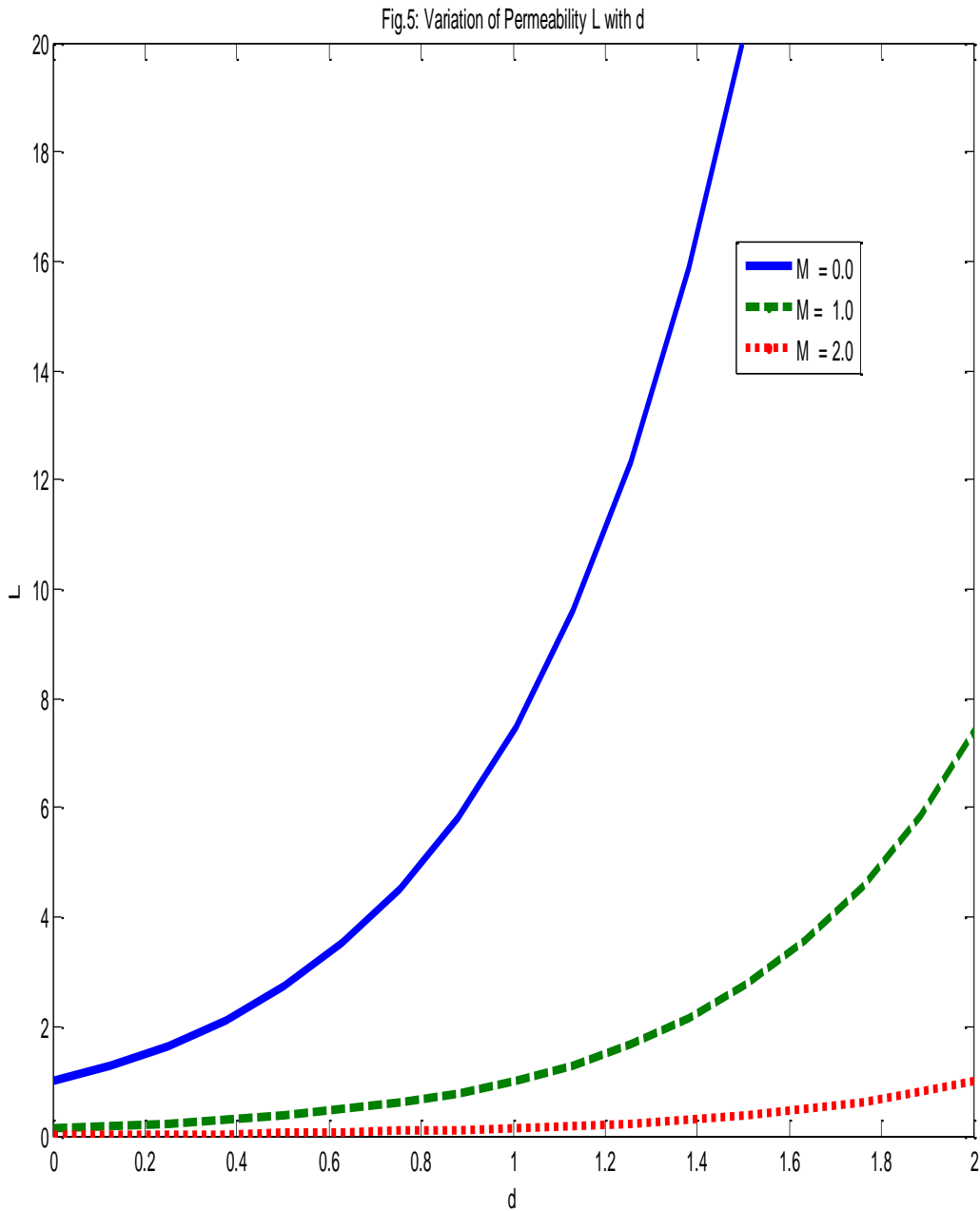
The effect of magnetic field on the velocity has been shown in Fig. 6. The velocity decreases with the increase of magnetic field and the velocity becomes constant at a fixed distance for a magnetic field.The variation in the graph can be visualized as we increase the magnetic field from 1.0 to 2.0 the effect on velocity is significant in comparison to the absence of magnetic field. As distance from origin increases the velocity increases smoothly.
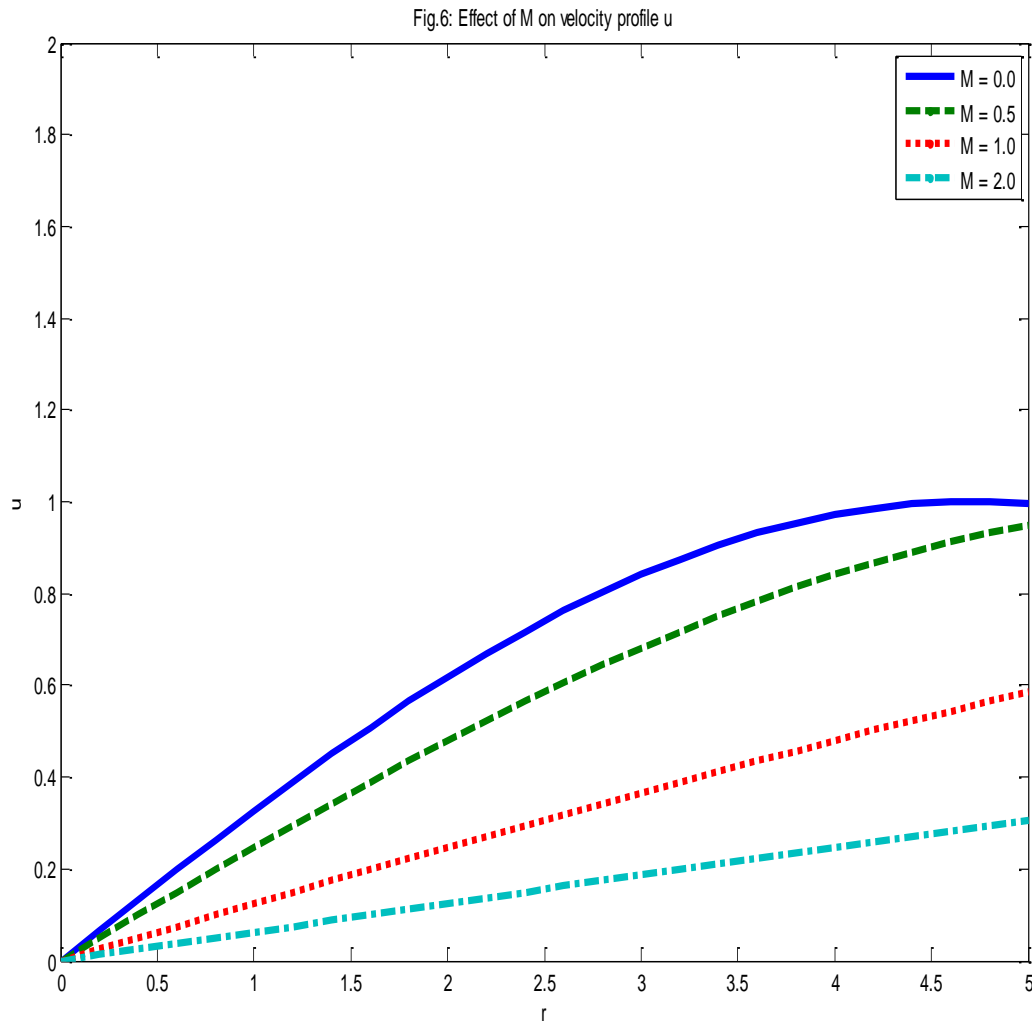
Fig.6: Effect of M on velocity profile u

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